LETTERS TO THE EDITOR

To the Editor:

F. Concha and M. C. Bustos (Feb., 1987, p. 312) have recently presented an interesting Note concerning a modification of the Kynch theory of sedimentation. Although I recognize the value of the work performed, I would like to comment on some aspects of their paper.

One criterion taken into account to single out the admissible weak solutions of Eq. 9 in their paper is Lax's entropy condition for shocks (Lax, 1957 and 1960). This condition can be applied to systems, in which the settling rate and the solids flux density depend only on the solids concentration. In this case, a hyperbolic system is considered. The zones that are formed in a batch test are the settling zone and the compression zone. There is a discontinuity of solids concentration between the two zones, corresponding to the surface of the sediment that builds up on the bottom of the cylinder. In the settling zone, the sedimentation rate and the solids flux density depend only on the solids concentration. In the compression zone, however, the subsidence rate depends on the solids concentration as well as the squeeze transmitted by the upper layers due to their weight (Fitch, 1979 and 1983; Tiller, 1981; Font, 1986). This is also recognized by the authors. Consequently, in the whole system (settling zone + compression zone), the sedimentation rate does not depend only on the solids concentration. Therefore, Lax's entropy condition cannot be considered.

My comments about Figure 1 of their paper are as follows:

1) $f(\phi) = u_\infty \phi (1 - \phi)^{n+1}$ is plotted vs. ϕ in Figure 1 of this letter. According to the shape of this Figure and Kynch's theory, the variation of the pulp-supernatant interface height should be that indicated in Figure 2 of this letter (Kynch, 1952; Koss, 1977). Accordingly, characteristics of ϕ equal to 0.17, 0.19 and 0.25 cannot appear.

2) All the characteristics, except for $\phi = 0.14$ and $\phi = 0.23$, must begin from the origin. In Figure 1 of their paper there are some characteristics corresponding to intermediate values of ϕ between 0.14 and 0.23, which begin from the t axis.

3) According to the equation $u_s = u_\infty (1 - \phi)^{n+1}$, for $\phi = 0.14$, one deduces that $u_s = 9.059 \times 10^{-5}$ m/s. The value u_s, which can be calculated from the initial straight line of the pulp-supernatant interface in their Figure 1, is approximately 6.8×10^{-5} m/s. Therefore, the result presented in their Figure 1 is not logical, and does not agree with Eq. 19 in their paper.

Probably the values h and k in the numerical solution are not large enough to obtain a correct solution.

The initial curve in Figure 2 of their paper, which corresponds to the variation of the pulp-supernatant interface height at small values of time, is difficult to explain. Nevertheless, I do not understand how the authors have obtained the characteristics corresponding to $\phi > \phi_c$ when Eq. 19 in their paper is valid for $\phi < \phi_c$ only. Furthermore, in the compression zone there are no characteristics, because the subsidence rate does not depend only on the solids concentration. One can define lines of equal concentration or lines of equal subsidence rate, but never characteristic lines.

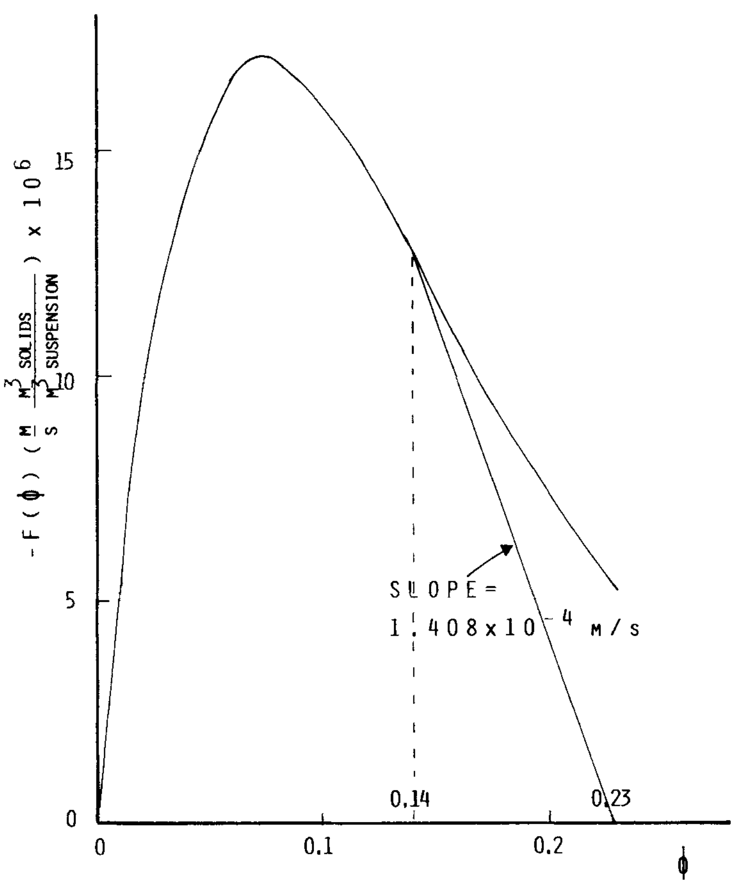

Figure 1. Solids flux density vs. solids concentration.

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R. Font
Div. of Chemical Engineering
University of Alicante
Alicante, Spain

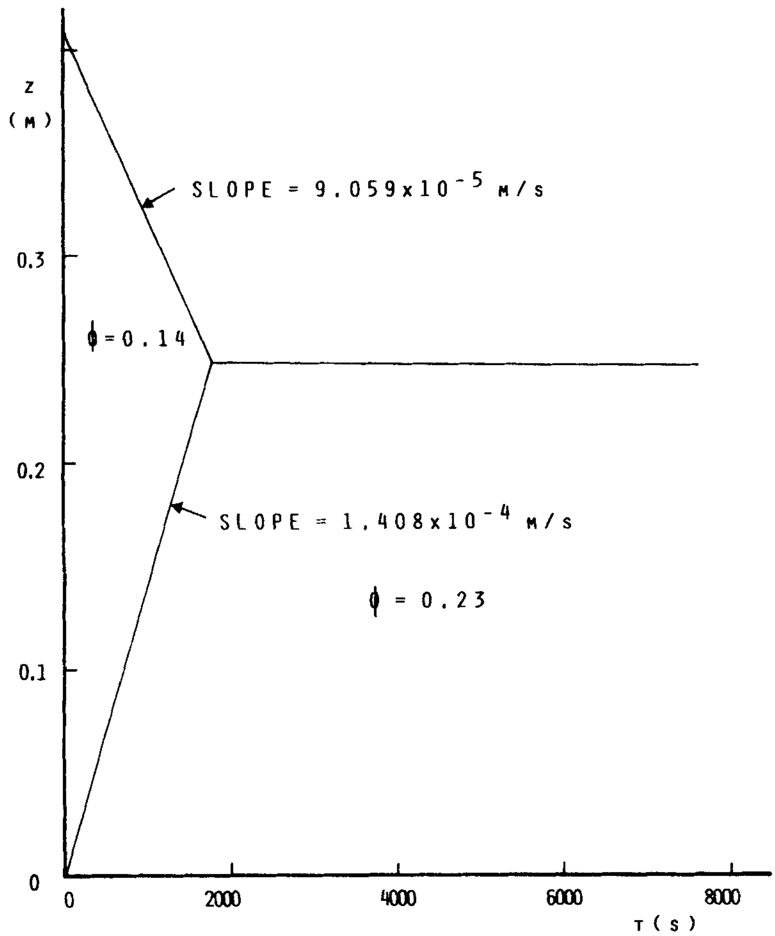

Figure 2. Height vs. time.

Reply:

We would like to thank Prof. Font for his comments and for pointing out some aspects of our paper that appear not to be clear.

First, we want to recall some fundamental mathematical concepts that we used in our paper. The Lax entropy condition is a general mathematical formulation that applies to all hyperbolic quasilinear conservation laws and not only to the problem of sedimentation, with or without compression, as Prof. Font suggests in his letter. The hyperbolic conservation laws consist in general of n equations in n unknown,

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial z} = 0,$$

where U and F are n vectors with components $U^T = (u_1, u_2, \ldots, u_n)$; $F^T = (f_1, f_2, \ldots, f_n)$. The characteristic speeds $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of the Jacobian matrix dF and, since the system is hyperbolic, all of them are different from one another. Corresponding to each characteristic speed, the system has n characteristic curves defined by (Courant and Hilbert, 1962):

$$\frac{dz}{dt} = \lambda_i(U), \quad i = \overline{1, n}.$$

Solutions to this type of equations are in general weak solutions that satisfy the n Rankine-Hugoniot conditions across the discontinuity $z(t)$:

$$\sigma = \frac{dz}{dt} = \frac{F(U^+) - F(U^-)}{U^+ - U^-}.$$

Across a shock of the kth family they also satisfy the Lax entropy condition (Lax, 1957):

$$\lambda_k(U^-) \geq \sigma \geq \lambda_k(U^+), \quad k = \overline{1, n}.$$

For $n = 1$, the characteristic speed $\lambda_1 = f'(u)$, so that the Lax entropy condition reduces to,

$$f'(u^-) \geq \sigma \geq f \pm (u^+).$$

With respect to Prof. Font's question on how to obtain the characteristic lines for concentrations greater than the critical concentration, we would like to point out that the objective of our paper is to describe the sedimentation of compressible suspensions within the framework of the Kynch theory of sedimentation (kinematical model). This means that we keep Kynch's equation (a conservation law with $n = 1$) and the initial condition, but we replace the boundary condition given in Eq. 5 for that given in Eq. 8 and extend the flux-density function $f(\phi)$ to values of $\phi > \phi_c$.

Since this procedure of including compression in the Kynch model appears to apply only to suspensions with very low compressibility, we developed a dynamical model (Bustos and Concha, 1986) in a second paper submitted to this *Journal.* This model consists of two hyperbolic quasilinear conservation laws with the concentration ϕ and the flux density f as unknowns. In both models, the Lax entropy condition should be applied to select the physical relevant solution.

We disagree with Prof. Font's assertion that in consolidating pulps no characteristics may be defined. For one hyperbolic quasilinear equation (Kynch kinematical model) or a system of two hyperbolic quasilinear equations (dynamical model), lines of equal concentration and characteristic curves can be defined (Courant and Hilbert, 1960). In the first case both coincide and in the second they do not. Prof. Font seems to forget that we are using Kynch model in the region of consolidating pulps in which the lines of equal concentration are its characteristics and are straight. This is the reason why we decided to develop a more general theory, since the experimental data available (Scott, 1968; Been and Sills, 1981) show that the lines of equal concentration in the consolidating pulp are curves with decreasing slope that become horizontal as time increases.

With respect to Figure 2, the strange shape of the initial part of the settling curve is due to a numerical problem we had at the time, in calculating the first values of the concentration at the

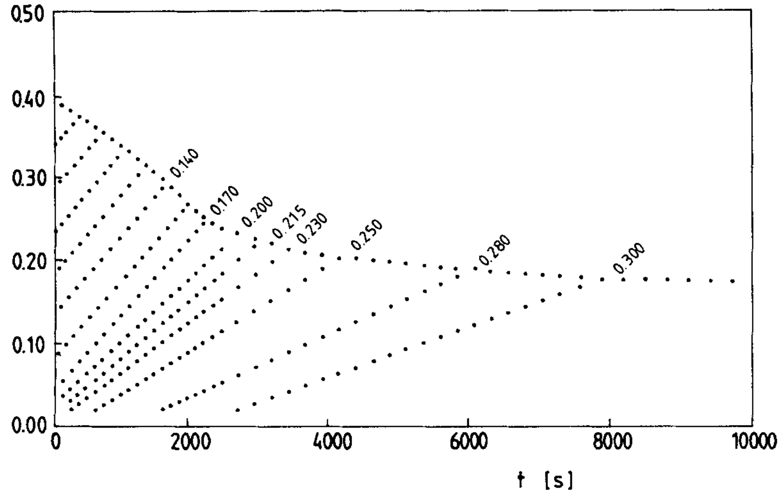

Figure 1. Modified Kynch theory: settling of a compressible suspension.

boundary. This gave rise to characteristics of concentration between 0.17 and 0.23 originating in the t axis. Since then, we have been able to overcome this difficulty as Figure 1 shows.

With respect to Figure 1 in our paper, we obtained the result that the consolidation of flocculated suspensions of low compressibility can be represented by straight lines diverging $\phi > \phi_c$. The purpose of including Figure 1 is to show the prediction of the Kynch model that the characteristics leaving the t axis are parallel for an ideal suspension. There is no claim in our paper that this solution represents the experimental results for compressible suspension; on the contrary, we have stressed that this is not the case. Furthermore, if we consider an ideal suspension, there would be no need to use a numerical method to solve Kynch problem, since the method of characteristics to construct global weak solutions is available (Bustos and Concha, 1987).

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F. Concha and M. C. Bustos
Dept. of Metallurgical Engineering
University of Concepcion
Concepcion, Chile